MATHEMATICAL.
OLYMPIAD.CH
MATHEMATIK-OLYMPIADE
OLYMPIADES DE MATHÉMATIQUES
OLIMPIADI DELLA MATEMATICA

IMO-Selection 2020

First exam
9 May 2020

Duration: 4.5 hours
Difficulty: The problems are orderd by difficulty.
Points: Each problem is worth 7 points.

1. Let $n \geq 2$ be an integer. Consider an $n \times n$ chessboard with the usual chessboard colouring. A move consists of choosing a 1×1 square and switching the colour of all squares in its row and column (including the chosen square itself). For which n is it possible to get a monochrome chessboard after a finite sequence of moves?

2. Find all positive integers n such that there exists an infinite set A of positive integers with the following property: For all pairwise distinct numbers $a_1, a_2, \ldots, a_n \in A$, the numbers

$$a_1 + a_2 + \ldots + a_n \quad \text{and} \quad a_1 \cdot a_2 \cdot \ldots \cdot a_n$$

are coprime.

3. Let k be a circle with centre O. Let AB be a chord of this circle with midpoint $M \neq O$. The tangents of k at the points A and B intersect at T. A line goes through T and intersects k in C and D with $CT < DT$ and $BC = BM$. Prove that the circumcentre of the triangle ADM is the reflection of O across the line AD.

Good Luck!

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Second exam
10 May 2020

Duration: 4.5 hours
Difficulty: The problems are orderd by difficulty.
Points: Each problem is worth 7 points.

4. Find all odd positive integers n such that for all pairs of positive coprime divisors a, b of n

$$a + b - 1 \mid n.$$

5. Find all polynomials Q with integer coefficients such that every prime number p and any two positive integers a, b with $p \mid ab - 1$ satisfy

$$p \mid Q(a)Q(b) - 1.$$

6. Prove that for every positive integer n, there exists a finite subset of the squares of an infinite chessboard that can be tiled with indistinguishable 1×2 dominoes in exactly n ways.

Good Luck!

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Third exam

23 May 2020

Duration: 4.5 hours
Difficulty: The problems are orderd by difficulty.
Points: Each problem is worth 7 points.

7. Find all functions $f\colon \mathbb{R} \to \mathbb{R}$ such that $0 \leq f(x) \leq 2x$ for all $x \geq 0$ and such that for all $x, y \in \mathbb{R}$

$$f(x+y) = f(x+f(y)).$$

8. Let I be the incenter of a non-isosceles triangle ABC. Let F be the intersection of the perpendicular to AI through I with BC. Let M be the point on the circumcircle of ABC such that $MB = MC$ and such that M is on the same side of the line BC as A. Let N be the second intersection of the line MI with the circumcircle of BIC. Prove that FN is tangent to the circumcircle of BIC.

9. We call a set S of integers *laikable* if for any positive integer n and any $a_0, a_1, \ldots, a_n \in S$, all integer roots of the polynomial $a_n x^n + \ldots + a_1 x + a_0$ are also in S, given that it is not the zero polynomial. Find all laikable sets of integers that contain all numbers of the form $2^a - 2^b$ for positive integers a, b.

Good Luck!

IMO-Selection 2020

Fourth exam

24 May 2020

Duration: 4.5 hours
Difficulty: The problems are orderd by difficulty.
Points: Each problem is worth 7 points.

10. Let ABC be a triangle with circumcircle k. Let A_1, B_1 and C_1 be points on the interior of the sides BC, CA and AB respectively. Let X be a point on k and denote by Y the second intersection of the circumcircles of BC_1X and CB_1X. Define the points P and Q to be the intersections of BY with B_1A_1 and CY with C_1A_1, respectively. Prove that A lies on the line PQ.

11. Let $a_0, a_1, a_2, \ldots$ be an infinite sequence of non-negative integers satisfying $a_i \leq i$ for every $i \geq 0$ and such that for every integer $n \geq 1$

$$\binom{n}{a_0} + \binom{n}{a_1} + \cdots + \binom{n}{a_n} = 2^n.$$

Prove that each non-negative integer appears in the sequence.

12. Let a, b, c, d be positive real numbers such that $a + b + c + d = 1$. Prove that

$$\left(\frac{a^2}{a+b} + \frac{b^2}{b+c} + \frac{c^2}{c+d} + \frac{d^2}{d+a}\right)^5 \geq 5^5 \left(\frac{ac}{27}\right)^2.$$

Good Luck!