1 Easy

Problem 1.

What is the smallest integer greater than 1 which is both a square and a cube?

Problem 2.

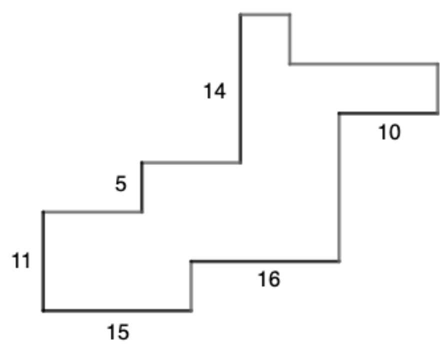
What is the smallest positive, odd integer that is not a prime and not a square?

Problem 3.

How many integers are there between 10 and 1000 that stay the same when the order of their digits is reversed?

Problem 4.

Assuming that all its angles are 90°, what is the perimeter of the following shape?



Problem 5.

The sum of five consecutive integers equals the sum of the three next bigger numbers. What is the biggest of these eight numbers?

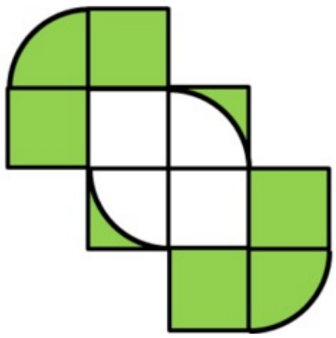
a) 4 b) 8 c) 9 d) 11 e) 12

Problem 6.

On a cube, we label each of its faces and vertices with 1 and each edge with -1. What is the total sum of the labels?

a) 0 b) 2 c) 4 d) 6 e) 8

Problem 7.



Using her compass, Rada drew this figure on a grid paper. If the side length of one small square is 2, what is the green area?

a) 12 b) 16 c) $16 + 2\pi$ d) 24 e) $24 + 2\pi$

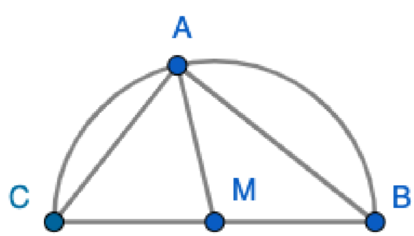
Problem 8.

Tim forgot the last five letters of his password. He only remembers that each letter is one of I, M or O. How many possibilities does he have to take into consideration?

a) 15 b) 20 c) 120 d) 125 e) 243

Problem 9.

Which statements about the following configuration must certainly hold, assuming that M is the center of the semicircle?



A $\measuredangle CAM = \measuredangle ABC$

B $\measuredangle AMC = 2\measuredangle ABC$

C $\measuredangle ACB + \measuredangle ABC = 90°$

D $Area(ACM) = Area(AMB)$

Problem 10.

The number 323 is...

A a prime

B a square

C the difference of two primes

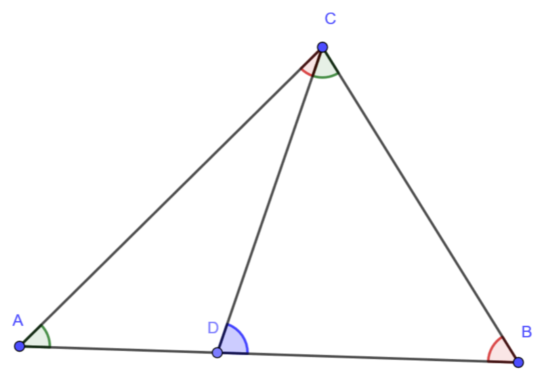
D the difference of two squares

2 Medium

Problem 11.

If the diagonal of square A is 16 times as large as the perimeter of square B, how many times as large is its area?

Problem 12.



Suppose that, in the diagram above, the angles $\measuredangle DAC$ and $\measuredangle DCB$ (both highlighted green) are $45°$ and that the angle $\measuredangle CBD$ is twice as large as the angle $\measuredangle ACD$ (both highlighted red). How many degrees is the blue angle $\measuredangle BDC$?

Problem 13.

Paul made less than 200 cookies and now wishes to distribute them into paper bags, such that each bag has exactly the same number of cookies. Sadly, this never seems to work out: If he wants to distribute them into 5 bags, there are four cookies left. The same thing happens if he wants to distribute them into 6 bags. If he wants to have 7 bags, there are 3 cookies left. How many cookies did Paul make in total?

Problem 14.

If a right angled triangle has a side of length 15 and a side of length 12, what is its smallest possible area?

Problem 15.

What integer has the property that if you square it and subtract 49 you get the same number as if you first subtract 49 and then square it?

Problem 16.

Henning's favourite ice cream store offers 8 different flavours. Henning wants to buy three scoops of ice cream which are not all of the same flavour. How many possibilities does he have to do this?
Remark: The order of the flavours doesn't matter.

Problem 17.

What is the remainder of $2^2 \times 3^3 \times 5^5 \times 7^7$ when divided by 8?

a) 2 b) 3 c) 4 d) 5 e) 7

Problem 18.

Given is a square $ABCD$ in the plane. How many squares are there that share exactly two vertices with $ABCD$?

a) 4 b) 6 c) 8 d) 12 e) 16

Problem 19.

Julia and Florian independently think of an integer between 1 and 10. Julia says: "No matter what number you chose: If we compute the product of our two numbers, it will not contain the digit 6.". Florian says: "Alright, then the sum of our numbers must be 14." What is Florian's number?

a) 4 b) 5 c) 6 d) 8 e) 9

Problem 20.

Which is the smallest integer $n > 2$ such that $(2^2 - 1) \cdot (3^2 - 1) \cdot \ldots \cdot (n^2 - 1)$ is a square number?

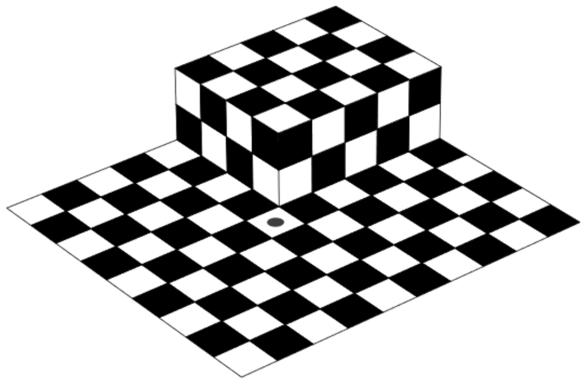
a) 7 b) 8 c) 9 d) 11 e) 12

Problem 21.

Tanish thinks of an integer $1 \leq n \leq 100$ and Marco wants to guess the number. After each guess, Tanish tells him whether his guess was correct, too small or too big. Marco wins as soon as he guesses the correct number. What is the smallest number of guesses that can guarantee Marco to win, assuming he guesses smartly?

a) 4 b) 5 c) 6 d) 7 e) 8

Problem 22.



An ant is initially on the square marked with the black dot. The ant moves across an edge from one square to an adjacent square four times and then stops. How many of the possible finishing squares are black?

a) 6 b) 8 c) 10 d) 12 e) 14

Problem 23.

If some positive integers a, b and c satisfy $a^2 + b^2 = c^2$, it follows that...

A $a + b > c$

B c is odd

C $a \neq b$

D c is not divisible by 7

Problem 24.

There are 51 distinct positive integers on the blackboard, none exceeding 100. Which statements about this board must be true?

A There are two consecutive numbers

B There are two numbers differing by 50

C There are two numbers summing to 100

D There are six numbers with the same last digit

Problem 25.

The expression $n^2 + n + 41$, for any positive integer n, is certainly...

A prime

B bigger than $(n+1)^2$

C odd

D not a square

3 Hard

Problem 26.

Quirin has 4 sticks of length 12. He breaks exactly one of them into two sticks and arranges all five pieces to a right-angled triangle. How big is the area of that triangle?

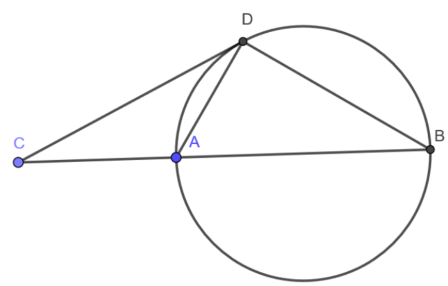
Problem 27.

Let n be the smallest positive integer, such that $10n$ is a square number and $6n$ is a cube number. What is n?

Problem 28.

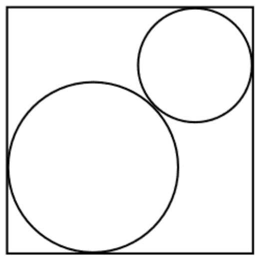
In how many different ways can a 2×11 rectangle be filled with 11 indistinguishable dominoes of size 1×2?

Problem 29.



AB is the diameter of a circle, C is on the line AB and CD is a tangent to the circle. If $|CA| = 34$, $|AD| = 48$ and $|DB| = 90$, what is the length of CD?

Problem 30.



Two circles are inscribed in a square of side length 1, as shown in the picture above. What is the sum of the two radii?

a) $\frac{1}{2}$

b) $\frac{1}{\sqrt{2}}$

c) $\sqrt{2} - 1$

d) $2 - \sqrt{2}$

e) The value depends on the circles.

Problem 31.

There are 100 coins in a line, all showing heads. Louis now turns every, then every second, then every third and so on up to every 100th coin. Assuming that the first coin was flipped every time, how many coins show heads at the end?

a) 50

b) 89

c) 90

d) 91

e) 99

Problem 32.

All the numbers from 1 up to 10 are written on the blackboard. David now repeatedly replaces two numbers by their (non-negative) difference until only one number is left. Which of the following could this number possibly be?

a) 0 b) 1 c) 4 d) 6 e) 11

Problem 33.

For a given positive integer $n > 1$, we write down all its positive divisors in ascending order:
$1 < d_1 < \ldots < d_k < n$. How many different n satisfy $d_k = 11 \cdot d_1$?

a) 0 b) 1 c) 2 d) 4 e) 5

Problem 34.

There exists a three-digit number ABC, such that

A ABC is divisible by C and the two-digit number AB

B ABC is divisible by A and the two-digit number BC

C $A > C > 0$ and $ABC - CBA$ is a prime number

D $ABC + BCA + CAB = 2021$

Problem 35.

The five friends A, B, C, D and E are playing a social deduction game. Good guys always have to tell the truth and bad guys always have to lie. This is their conversation:

A: "C and D are either both good or both bad."

B: "If E is good, A is telling the truth!"

C: "There is an even number of bad guys in the game."

D: "At least one among A, B and C has to be bad."

E: "A and C are not both good."

Which of the following statements are true?

A It is possible that A and C are both good

B D is certainly good

C B is certainly bad

D It's possible that there is only one good player